Code: 23BS1101

**I B.Tech - I Semester – Regular Examinations - JANUARY 2024**

# LINEAR ALGEBRA & CALCULUS

**(Common for ALL BRANCHES)**

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

## PART – A

| | | BL | CO |
|---|---|---|---|
| 1.a) | Estimate the value of $a$ , if the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & a & 4 \\ 1 & -1 & 1 \end{bmatrix}$ is 2 | L2 | CO1 |
| 1.b) | If the initial approximation to the solution of $10x+2y+z=9,\ 2x+20y-2z=-44,\ -2x+3y+10z=22$ is $(x,y,z)=(0,0,0)$ then find the first approximation by using Gauss-Seidel iteration method. | L3 | C04 |
| 1.c) | If the eigen values of $A=\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are 2, 3 & 6 then predict the eigen values of $A^{-1}$. | L2 | CO2 |
| 1.d) | Write down the quadratic form $X^T AX$ corresponding to the symmetric matrix $A=\begin{bmatrix} 1 & 3 & -5 \\ 3 & 2 & 0 \\ -5 & 0 & -4 \end{bmatrix}$ | L2 | CO4 |
| 1.e) | Discuss the applicability of Cauchy's mean value theorem for $f(x) = \begin{cases} -x\ , & if -4 < x < 0 \\ x\ , & if \ \ 0 \le x < 4 \end{cases}$ and $g(x) = x^2$ in $[-4, 4]$ | L2 | CO3 |

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|---|---|---|---|
| 1.f) | State the Maclaurin's series expansion of $f(x)$ about $x=0$. | L1 | CO3 |
| 1.g) | Estimate $\lim\limits_{\substack{x\to 1\\ y\to 2}} \frac{2x^2 y}{x^2+y^2+1}$ | L2 | CO1 |
| 1.h) | Estimate the first and second order partial derivatives of $f(x,y)=ax^2+2hxy+by^2$ | L2 | CO1 |
| 1.i) | Write the limits by changing the order of integration of the double integral $\int_0^1 \int_y^{y^2} (x+y)\ dxdy$ with the help of region of integration. | L2 | CO5 |
| 1.j) | Calculate the double integral $\int_0^1 \int_0^1 xy\ dydx$. | L3 | CO5 |

## PART – B

| | | | BL | CO | Max. Marks |
|---|---|---|---|---|---|
| **UNIT-I** | | | | | |
| 2 | a) | Discover the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & -3 & -3 \\ 1 & 0 & 1 & 2 \\ 1 & -1 & 3 & 3 \end{bmatrix}$ by reducing the matrix to Echelon form. | L3 | CO2 | 5 M |
| | b) | Solve the system of non-homogeneous linear equations $5x_1+3x_2+7x_3=4$, $3x_1+26x_2+2x_3=9$ and $7x_1+2x_2+10x_3=5$ | L3 | CO2 | 5 M |
| **OR** | | | | | |
| 3 | a) | Apply Gauss Jordan method to find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ | L3 | CO2 | 5 M |
| | b) | Make use of Jacobi's method to find first five iterations of the following system of equations $20x+y-2z=17$, $3x+20y-z=-18$, $2x-3y+20z=25$ | L3 | CO2 | 5 M |

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| **UNIT-II** | | | | | |
| 4 | a) | Calculate the characteristic roots and characteristic vectors of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ | L3 | CO2 | 5 M |
| | b) | Make use of the eigen values of matrix of the quadratic form to discuss the rank and nature of the quadratic form $-x_1^2-4x_2^2-x_3^2+4x_1x_2-4x_2x_3-2x_1x_3$ | L4 | CO4 | 5 M |
| **OR** | | | | | |
| 5 | a) | Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and hence find $A^4$. | L3 | CO2 | 5 M |
| | b) | Use Diagonalization to find the matrix *A*, if the eigen values of a matrix A of order 3 and the corresponding eigen vectors are 0,3,15 & $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ respectively. | L3 | CO2 | 5 M |
| **UNIT-III** | | | | | |
| 6 | a) | Check the applicability of Rolle's theorem, if applicable verify theorem for the function $\log\left\{\frac{x^2+ab}{x(a+b)}\right\}$ in $[a,b]$, where $0<a<b$ | L3 | CO5 | 5 M |
| | b) | Construct the series expansion of $f(x)=\log(1+x)$ in powers of $x$ up to third degree terms. | L3 | CO5 | 5 M |
| **OR** | | | | | |
| 7 | a) | Apply mean value theorem to prove that $\frac{b-a}{1+b^2} < tan^{-1} b - tan^{-1} a < \frac{b-a}{1+a^2}$ $(0<a<b)$ and hence deduce that $\frac{\pi}{4}+\frac{3}{25} < tan^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$. | L3 | CO5 | 5 M |

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| | b) | Discover the series expansion of $f(x) = Sin\, x$ in powers of $x - \frac{\pi}{4}$ | L3 | CO5 | 5 M |
| **UNIT-IV** | | | | | |
| 8 | a) | Point out the functions $u = x\, e^y sin\, z,\ \ v = x\, e^y cos\, z,\ w = x^2 e^{2y}$ are functionally dependent or not. If functionally dependent, find the relation between them. | L3 | CO5 | 5 M |
| | b) | Discover the nature of stationary points and then find extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ | L3 | CO3 | 5 M |
| **OR** | | | | | |
| 9 | a) | Make use of functional determinant to show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3 \sin 2\theta$ where $u = x^2 - 2y^2, v = 2x^2 - y^2$ and $x = r\cos\theta, y = r\sin\theta$ | L3 | CO5 | 5 M |
| | b) | Divide twenty-four into three parts such that the continued product of the first part, square of the second part and the cube of third part is maximum. | L4 | CO3 | 5 M |
| **UNIT-V** | | | | | |
| 10 | a) | By changing the order of integration, evaluate the double integral $\int_0^2 \int_{e^x}^{e} \frac{1}{log\, y}\, dy\, dx$ | L3 | CO5 | 5 M |
| | b) | Calculate the volume of the solid bounded by the planes $x = 0, y = 0, z = 0\ and\ x + y + z = 1.$ | L3 | CO3 | 5 M |
| **OR** | | | | | |
| 11 | a) | Calculate the triple integral $\int_{-1}^{1} \int_0^2 \int_1^3 x^2 y^2 z^3\, dx\, dy\, dz.$ | L3 | CO5 | 5 M |
| | b) | Discover the area enclosed by the pair of curves $y^2 = x$ and $y = x^2$ using double integration. | L3 | CO3 | 5 M |